

STANDARD FORM

Summary:

1. Standard form is a way of expressing a number in the form $A \times 10^n$ where $1 \leq A < 10$ and n is an integer
2. (i) To express a number in standard form, we shift the decimal point until the digit part A is between 1 and 10. This digit A has a decimal point placed after the first digit.

(ii) The power part (10^n) shows how many places to move the decimal point
3. The rules of indices apply to calculations in standard form

EXAMPLES:

1. Express the following numbers in standard form:

- (i) 25000 (ii) 3860 (iii) 568.3 (iv) 74.8 (v) 3.584 (vi) 4000
(vii) 0.435 (viii) 0.000263 (ix) 0.007 (x) 0.00356 (xi) 248×10^3
(xii) 4500×10^{-7} (xiii) 58.4×10^{-4} (xiv) 0.027×10^{-3}
(xv) 0.0062×10^{-4} (xvi) 0.0364×10^5

2. By expressing each of the numbers in standard form, evaluate the following:

- (i) 0.0004×0.002 (ii) 0.005×0.00004 (iii) 800000×0.0005
(iv) $\frac{800}{0.004}$ (v) $\frac{0.00006}{2000}$ (vi) $\frac{0.0009 \times 8000}{0.002 \times 0.3}$

3. By expressing each of the numbers in the form $a \times 10^n$ where n is even, evaluate the following:

(i) $\frac{0.81}{0.0027}$ (ii) 0.02×0.0015 (iii) 1500×40000

4. By expressing each of the numbers in the form $a \times 10^n$ where n is even, find the square root of:

(i) 0.25×0.64 (ii) 0.16×0.09 (iii) 0.0036×0.25 (iv) 0.0081×0.0004

5. Without using a calculator, evaluate the following give your answer in standard form:

(i) $(2 \times 10^2) \times (6 \times 10^5)$ (ii) $(4 \times 10^{-3}) \times (5 \times 10^4)$
 (iii) $(9 \times 10^{-2}) \times (4 \times 10^{-3})$ (iv) $(15 \times 10^{-3}) \times (20 \times 10^7)$

6. Without using a calculator, evaluate: $\frac{1.21 \times 10^{-2} \times 40}{2.2 \times 11}$

7. Without using a calculator, evaluate: $\frac{2.25 \times 10^{-3} \times 3.9}{1.3 \times 1.5}$

EER:

1. Without using a calculator, evaluate: $\frac{0.0035}{0.07 \times 0.2}$

2. Without using a calculator, evaluate: $\frac{0.625 \times 0.009}{0.0045}$

3. Without using a calculator, evaluate: $\frac{1.69 \times 10^{-2} \times 1.25}{1.3 \times 2.5}$

4. Without using a calculator, evaluate: $\frac{5600}{80000}$ give your answer in the form $a \times 10^n$ where $1 \leq a < 10$ and n is an integer

LOGARITHMS

Summary:

1. The logarithm of x to the base a is written as $\log_a x$

2. The following rules apply to logarithm:

$$(i) \log_c a + \log_c b = \log_c ab$$

$$(ii) \log_c a - \log_c b = \log_c (a/b)$$

$$(iii) \log_c a^m = m \log_c a$$

$$(iv) \log_c c = 1$$

$$(v) \log_c 1 = 0$$

$$(vi) \log_c a = b \text{ implies that } a = c^b$$

$$(vii) \log_c a = \log_c b \text{ implies that } a = b$$

$$(viii) \log_{10} a \text{ is sometimes written as } \log a$$

EXAMPLES:

1. Without using a calculator simplify:

$$(i) \log_{10} 10$$

$$(ii) \log_{10} 100$$

$$(iii) \log_{10} 1000$$

$$(vi) \log_{10} 10000$$

$$(v) \log_2 16$$

$$(vi) \frac{1}{2} \log_2 64$$

$$(vii) \log_5 125$$

$$(viii) \frac{1}{3} \log_3 729$$

$$(vii) \log_3 81$$

2. Without using a calculator simplify:

$$(i) \log_{10} 40 + \log_{10} 50 - \log_{10} 20$$

$$(ii) 2\log_{10} 50 + \log_{10} 80 - \log_{10} 2$$

$$(iii) \log_2 56 - \frac{1}{2} \log_2 49$$

$$(iv) \log 75 + 2\log 2 - \log 3$$

$$(v) 4\log_{10} 5 + \frac{1}{2} \log_{10} 64 - \frac{1}{3} \log_{10} 125$$

$$(vi) \log_{10} 5 - \frac{1}{2} \log_{10} 16 - \frac{3}{2} \log_{10} 25$$

$$(vii) \log_2 18 + \log_2 32 - 2\log_2 6$$

$$(viii) \frac{1}{2} \log_{10} 16 + \log_{10} a^2 - 2\log_{10} (a/5)$$

$$(ix) \frac{2}{3} \log 125 + \frac{1}{2} \log 49 - \log (7/4)$$

$$(x) \log_a a + 2\log_a a - \log_a a^{-3}$$

$$(xi) \frac{\log 8}{\log 2 + \log 16}$$

$$(xii) \frac{1}{4} + \frac{\log 27}{\log 9 + \log 81}$$

$$(xiii) \frac{\log 125}{\log 25} - \frac{\log 8}{\log 32}$$

3. Express the following as a single logarithm:

$$(i) 2\log_{10} 3 + \frac{1}{3}\log_{10} 216 + \frac{1}{2}\log_{10} 16 - \log_{10} 12$$

$$(ii) 2\log_{10} 8 + \frac{1}{3}\log_{10} 125 - 1$$

$$(iii) 3\log_3 12 + \log_3 3^{-1} - \log_3 6^2 + 1$$

$$(iv) \log_{10} 120 + \frac{1}{2}\log_{10} 25 - 2$$

4. Solve the simultaneous equations

$$\log_3 (2x + y) = 2$$

$$\log_2 (5x + 3y) = 2$$

5. Solve for x in the equations:

$$(i) 2\log x + \log 6 = 1 + \log 15$$

$$(ii) \log_{10} (3x + 1) - \log_{10} (x - 1) = \log_{10} 4$$

$$(iii) \log_{10} (x + 1) + \log_{10} (x - 2) = \log_{10} 10$$

$$(vi) \log_4 (x + 3) - \log_4 (x - 3) = 2$$

6. Use logarithm tables to write down the logarithm of the following numbers:

$$(i) 1826$$

$$(ii) 25$$

$$(iii) 6.25$$

$$(iv) 37.28$$

$$(v) 0.0812$$

$$(vi) 0.2147$$

$$(vii) 0.0025$$

$$(viii) 0.0007$$

7. Use the fact that $\log_{10} 3 = 0.4771$ and $\log_{10} 0.3 = \bar{1}.4771$ to find:

$$(i) \log_{10} 3^2$$

$$(ii) \log_{10} 3^3$$

$$(iii) \log_{10} \sqrt{3}$$

$$(iv) \log_{10} \sqrt[3]{3}$$

$$(v) \log_{10} (0.3)^2$$

$$(vi) \log_{10} (0.3)^4$$

$$(vii) \log_{10} \sqrt{0.3}$$

$$(viii) \log_{10} \sqrt[3]{0.3}$$

$$(ix) \log_{10} (0.3)^{2/3}$$

8. Use logarithm tables to write down the logarithm of the following numbers:

$$(i) (24 \cdot 68)^2$$

$$(ii) (195 \cdot 6)^3$$

$$(iii) \sqrt{72 \cdot 2}$$

$$(iv) \sqrt[3]{50 \cdot 37}$$

$$(v) (7 \cdot 38)^{2/3}$$

$$(vi) (0.567)^2$$

$$(vii) (0.0248)^3$$

$$(viii) \sqrt{0.0547}$$

$$(ix) \sqrt{0.5385}$$

$$(x) \sqrt[3]{0.472}$$

$$(xi) \sqrt[5]{0.0843}$$

$$(xii) (0.48)^{2/3}$$

9. Use the fact that $\log x = 0.4771$, $\log y = 0.8451$ and $\log z = \bar{1}.3010$ to

find:

(i) $\log xy$

(ii) $\log xz$

(iii) $\log yz$

(iv)

$\log x/y$

(v) $\log y/x$

(vi) $\log x/z$

(vii) $\log z/x$

(viii)

$\log y/z$

(ix) $\log z/y$

(x) $\log xy/z$

(xi) $\log x/yz$

(xii)

$\log x^2 y^3$

(xiii) $\log yz^4$

(xiv) $\log xz^{\frac{2}{3}}$

(xv) $\log \sqrt[3]{yz}$

(xvi)

$\log x^{\frac{1}{3}} z^{\frac{1}{2}}$

10. Use the fact that $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$ to find:

(i) $\log_{10} 6$

(ii) $\log_{10} 60$

(iii) $\log_{10} 72$

(iv) $\log_{10} 5$

(v) $\log_{10} 150$

(vi) $\log_{10} 6.4$

(vii) $\log_{10} 0.72$

(viii) $\log_{10} 1.5$

11. Use logarithm tables or a calculator to find the value of x for which:

(i) $\log_{10} x = 0.7781$

$$(ii) \log_{10} x = 1.3010$$

$$(iii) \log_{10} x = 1.6020$$

$$(iv) \log_{10} x = \bar{1}.4771$$

$$(v) \log_{10} x = \bar{2}.1761$$

12. Use the fact that $\log 7 = 0.8451$ and $\log 2 = 0.3010$ to find:

(i) $\log(64/49)$, hence obtain the value of $\frac{64}{49}$ correct to **3** decimal places.

(ii) $\log(49/64)$, hence obtain the value of $\frac{49}{64}$ correct to **3** decimal places.

13. Given that $\log x = 2.304$ and $\log y = \bar{2}.872$ find the value of:

(i) $\log x^{1/2}/y$, hence find the value of $x^{1/2}/y$ correct to **3** decimal places.

(ii) $x^{\frac{1}{4}} y^{-\frac{1}{3}}$ correct to **one** significant figure.

14. Given that $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, without using tables or a

calculator, find the value of x for which:

$$(i) \log_{10} x = 0.7781$$

$$(ii) \log_{10} x = 0.1761$$

$$(iii) \log_{10} x = 1.4771$$

$$(vi) \log_{10} x = 1.3010$$

$$(v) \log_{10} x = 1.6020$$

15. Use logarithm tables to evaluate the following:

$$(i) 25.6 \times 731$$

$$(ii) \frac{22.6}{47.8}$$

$$(iii) \frac{328 \times 0.3465}{46.82}$$

$$(iv) \frac{22.6}{47.8 \times 0.329}$$

$$(v) \frac{0.0721 \times 925}{0.00322 \times 405}$$

$$(vi) 5.6 \times (14.73)^2$$

$$(vii) \frac{3.83 \times 5.968}{(1.597)^2}$$

$$(viii) \frac{\sqrt{3.37} \times 0.429}{76.51}$$

$$(ix) \sqrt{\frac{32.6}{17.86}}$$

$$(ix) \sqrt[3]{(0.3256)^2}$$

$$(x) \sqrt[3]{\frac{0.3216 \times 62.58}{41.57}}$$

$$(xi) \left(\frac{24.8 \times 205}{0.04763} \right)^{2/3}$$

16. Solve for x in the following equations:

$$(i) 5^x = 9$$

$$(ii) 3^x = 5$$

(iii) $7^x = 15$

(iv) $3^x = 12$

(v) $1 \cdot 2^x = 1.728$

17. Make n the subject in this formula $A = P\left(1 + \frac{r}{100}\right)^n$, hence find n when

$r = 20, P = 2000$ and $A = 3456$.

18. A sum of **Sh. 400,000** is invested in a bank that gives **15%** compound interest

per annum. Find how long it will take to accumulate an amount of **sh529,000**.

EER:

1. Without using a calculator, simplify:

$$(i) \log_{10} 24 + 2\log_{10} 5 - \log_{10} 6$$

$$(ii) \log 12 + \log 5 - \log 6$$

$$(iii) \log_{10} 5 - \frac{1}{2}\log_{10} 16 - \frac{3}{2}\log_{10} 25$$

$$(iv) \log_{10} 20 + \frac{1}{3}\log_{10} 125$$

$$(v) \log_2 18 + \log_2 32 - 2\log_2 6$$

$$(vi) \frac{1}{2}\log_{10} 16a^6 + 2\log_{10} 5 - \frac{3}{2}\log_{10} a^2$$

$$(vii) 3\log_{10} 5 + 6\log_{10} 2 - \frac{1}{2}\log_{10} 64$$

$$(viii) \frac{\log 343}{\log 49} - \frac{\log 8}{\log 32}$$

$$(ix) \frac{2}{3}\log 125 + \frac{1}{2}\log 49 - \log (7/4)$$

$$(x) \frac{5}{2}\log_a a + \frac{1}{2}\log_a a^9 - 3\log_a a$$

2. Express the following as a single logarithm:

$$(i) 2\log_{10} 6 + \frac{3}{2}\log_{10} 9 - 4\log_{10} 3$$

$$(ii) \log_{10} 48 + \frac{1}{2}\log_{10} 25 - 1$$

$$(ii) \log_{10} 120 + \frac{1}{2}\log_{10} 25 - 2$$

3. (i) Find the prime factors of 150. Hence

(ii) find $\log_{10} 150$, given that $\log_{10} 5 = 0.6690$, $\log_{10} 3 = 0.4771$ and

$$\log_{10} 2 = 0.3010$$

4. Solve for x in the equation:

(i) $2\log x + \log 2 = \log 18$

(ii) $\log_{10} (2x + 4) - \log_{10} (1 - x) = 1$

(iii) $4\log \frac{x}{4} - \log \frac{2x}{4} = 1$

(iv) $\log \frac{x}{8} + \log \frac{2x}{8} = 1$

(v) $2\log \frac{(x-1)}{16} - 2\log \frac{5}{16} = \frac{1}{2}$

(vi) $\log_{12} (x+9) - \log_{12} (x-9) = 2$

5. Use the substitution $y = \log_{10} x$ to solve for x in the equation:

$$\left(\log_{10} x \right)^2 - 4\log_{10} x + 4 = 0$$

6. Use the fact that $\log_{10} 2 = 0.3010$ and $x = 4$ to find the value of $\log_{10} x^2$.

7. Given that $5^x = 7$, find the value of x correct to **two** decimal places

8. Make n the subject in this formula $T = ar^{n-1}$, hence find n when

$$r = 5, a = 0.48 \text{ and } T = 60.$$

9. A sum of **Sh. 400,000** is invested in a bank that gives **15%** compound interest

per annum. Find how long it will take to accumulate an amount of **sh529,000**.

10. A sum of **Sh. 200,000** is invested in a bank that gives **20%** compound interest

per annum. Find how long it will take to accumulate an amount of **sh345,600**.

11. Given that $\log a = 0.234$ and $\log b = 1.185$, find $\log a^5 b^2$.

12. Use the fact that $\log x = 0.648$ and $\log y = 1.913$, to find $\log xy^{\frac{1}{3}}$.

13. Given that $\log_{10} x = 2.852$ and $\log_{10} y = 2.581$, use tables to evaluate

$$x^{\frac{1}{2}}/y \text{ correct to **three** significant figures}$$

14. Given that $\log_{10} p = 2.476$ and $\log_{10} q = 1.811$, find $\log_{10} (p/q^2)$.

15. Use the fact that $\log_{10} 2 = 0.3010$, to find $\log_{10} 5$

16. (i) Show that $\log_{10} 1 = 0$

(ii) Without using tables or a calculator, find $\log_{10} 0.001$

17. Use the fact that $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$ to find:

(i) $\log_{10} 12$

(ii) $\log_{10} 80$

(iii) $\log_{10} 240$

(iv) $\log_{10} (27/16)$

(v) $\log_{10} 5$

(vi) $\log_{10} 15$

(vii) $\log_{10} 2.5$

(viii) $\log_{10} 1.8$

18. Use the fact that $\log_{10} 7 = 0.8451$ and $\log_{10} 6 = 0.7782$ to find
 $\log_{10} 25 \cdot 2$.

19. Use the fact that $\log_{10} 32.5 = 1.5119$ to find:

(i) $\log_{10} 32500$

(ii) $\log_{10} 0.00325$

(iii) $\log_{10} 3.25$

20. Use the fact that $\log_{10} 0.225 = \bar{1}.3522$ to find:

(i) $\log_{10} \sqrt{0.225}$

(ii) $\log_{10} \sqrt[3]{0.225}$

21. Find x if:

(i) $7^x = 15$

(ii) $\log_{10} x = 1.3472$

(iii) $1.4^x = 2.744$

(iv) $\log_{10} x = \bar{2}.4865$

22. Given that $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, without using tables or a

calculator, find the value of:

(i) $\log_{10} 0.6$

(ii) $\log_{10} 1.5$

(iii) x if $\log_{10} x = 1.9030$

23. Given that $x^3 = 3.375$, use tables to find the value of x correct to three

significant figures

24. Use logarithm tables to evaluate the following:

(i) 75.6×0.8563

(ii) $\frac{12.75}{7.908}$

(iii) $\frac{12.75 \times 28.34}{7.908}$

(iv) $\frac{925}{0.00322 \times 405}$

(v) $\frac{0.0075 \times 986}{0.04562 \times 225}$

(vi) $(18.26)^4$

(vii) $\frac{5.6 \times (14.73)^2}{34.26}$

(viii) $\frac{22.6 \times 3.657}{(0.329)^4}$

(ix) $\sqrt{32.83}$

$$(ix) \sqrt[3]{11.86}$$

$$(x) (0.48)^{3/5}$$

$$(xi) \sqrt{(31.35)^3}$$

$$(xii) \sqrt{\frac{0.0722}{0.00916}}$$

$$(xiii) \frac{\sqrt{3.83} \times 5.968}{(1.597)^2}$$

$$(xiv) \frac{9.324 \times 0.3639}{\sqrt[3]{81.95}}$$

$$(xv) \sqrt[3]{\frac{0.3215 \times 1.439}{0.00485}}$$

$$(xvi) \sqrt[3]{\left(\frac{12.75 \times 28.34}{7.908}\right)^2}$$

$$(xvii) \frac{19.76}{(8.27)^3}$$

$$(xviii) \frac{0.56}{\sqrt{0.08356}}$$